2013 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

1. Let k be the number of distinct diagonals that can be drawn in a dodecagon. Let w be the numeric value of the average degree measure of an exterior angle of a dodecagon. Find the value of $(k+w)$.

2. Let A represent the smallest product of two real numbers whose difference is 6. Let B represent the largest value of y such that $y=-|x+6|+3$ where x is a real number. Find the value of the product AB.

3. A triangle and a square both have a perimeter of 2013. The sides of the triangle are in the ratio 5:11:13. Let k be the average length of a side of the triangle. Let w be the length of the diagonal of the square. Find the value of $(k+w)$ rounded to the nearest integer.

4. Let x, y, and z be consecutive odd integers such that $x>y>z$. Find the value of $(x-y)(y-z)(x-z)$.

5. In base x, $123_x=227_{ten}$. In base y, $147_y=628_{ten}$. Find the value of $(x+y)$ as a base ten numeral if both x and y are positive integers.

6. Let k be the number of distinct positive integers that leave a remainder of 23 when divided into 1904. Let S be the sum of the first eleven terms of the geometric progression whose first term is 1 and whos fourth term is 8. Find the value of $(k+S)$

7. From the set {8, 9, 18, 27, 36, 54, 81} two numbers are selected at random. Find the probability that at least one of the numbers selected is one of the terms of the geometric sequence: $\frac{2}{27},\frac{2}{9},\frac{2}{3},\ldots$. Express your answer as a fraction reduced to lowest terms.

8. In the game of "seven come eleven", a player can only score 7 points or 11 points if (s)he makes a shot. Find the largest integer score that is unattainable regardless of the number of shots made.

9. (T1) Let $i=\sqrt{-1}$ and let x and y be real numbers. If $216i+\log(x)-1=3^y i-27i$, find the value of $(x+y)$

10. (T2) ΔABC is isosceles with perimeter 306. Two sides have lengths $(4x+8)$ and $(3x+4)$. Find the sum of the two possible values for x.

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Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

1. **84**

2. **–27**

3. **1383**

4. **16**

5. **37**

6. **2054**

7. **11/21** **Must be this fraction**

8. **59**

T1. **15**

T2. **55**

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

Note: Questions (9) and (10) will be used to break ties for positions 1, 2, and 3. If a tie remains after two tiebreaker questions, the tie will be broken if possible using (a) Total # correct answers, (b) Total # firsts, (c) student total scores on the individual exams.